COMPRESSED SENSING AND MINIMAX DENOISING

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ABSTRACT

A sparse vector can be regarded as a collection of samples from a sparse mixture distribution, defined as a mixture of a delta function located at zero and an arbitrary probability distribution. l_1 reconstruction provides an efficient way to reconstruct such a sparse vector from linear measurements on it. Critical compression rate for l_1 reconstruction is defined as the critical value of the ratio of the number of measurements to the dimension of the sparse vector, above which l_1 reconstruction succeeds and below which it fails in the large-system limit, i.e., in the limit where the number of measurements and the dimension are sent to infinity while their ratio is kept finite. In this talk, we describe critical compression rates as a quantity having a role similar to the Shannon entropy. This description is based on our study of the case where one knows prior probabilities for the elements of the sparse vector to take non-zero values. We have studied how to optimize weights in weighted l_1 reconstruction, and have shown that critical compression rates are additive if the weights of l_1 reconstruction are optimally chosen. The relation between critical compression rates and minimax risk of Bayesian denoising in Gaussian channels is also reviewed.